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NASA TECHNICAL MEMORANDUM

REPORT NO. 53912

ELECTRON TRANSPORT FROM A COSINE LAW SOURCE

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September 16, 1969

NASA

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MSFC - Form 3190 (September 1968)

INTERNAL NOTE SSL-N-68-13

Changed to TM X-53912, September 16, 1969

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ABSTRACT

This report presents a technique for calculating the electron transport and dose expected behind aluminum shields. The method of analysis was first used by Burrell, et al. (NASA SP-169). This work includes results for a source whose angular distribution of flux incident on a shield follows the cosine function. Included is a comparison of the dose expected from a cosine law source to doses from sources of isotropic and normal angular distribution.

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RESEARCH AND DEVELOPMENT OPERATIONS

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INTRODUCTION

The purpose of this report is to extend the computational methods of Burrell, Wright, and Watts [1] for the calculation of electron transport in aluminum shields. The original work considered electron transport for cases in which the electron flux was either normally or isotropically incident on a shield. It is possible to represent reality more closely be representing the flux of incident electrons as following a cosine law distribution. This report includes predictions of the electron dose rate expected behind spacecraft shields for each source distribution, normal, isotropic, and cosine law.

INTERPRETATION OF ELECTRON MONTE CARLO DATA

To compare this work with previous calculations, it is necessary to review the methodology already established. Berger [2] presented electron energy transfer data computed by Monte Carlo methods. These data gave electron energy and number factors for transmission and reflection at five energies between 0.125 and 2.0 MeV and at five angles between 0 and $\pi/2$ as a function of reduced shield thickness $X = Z/r_0$, where Z is the depth normal to the surface and r_0 is the mean range of an electron of energy E_0. The energy transmission and reflection factors are defined by Berger as follows:

$$T_E(X,\theta) = \frac{\text{energy transmitted at } X}{\text{energy incident on shield at angle } \theta} \tag{1}$$

$$B_E(X,\theta) = \frac{\text{energy reflected at } X}{\text{energy incident on shield at angle } \theta} \tag{2}$$

The number transmission and reflection coefficients T_N and B_N are similarly defined in terms of the number of electrons transmitted or reflected.

Particles incident on a shield can penetrate it, be reflected back out of it, or be absorbed between the surface and the shield thickness, X. Assuming conservation of particles, the sum of the probabilities for the occurrence of each of these events is 1. Thus, the energy or number current is:

$$1 = T_E(X,\theta) + B_E(X,\theta) + \int_0^X P(X',\theta)\,dX' \,. \tag{3}$$

$$1 = T_N(X,\theta) + T_N(X,\theta) + \int_0^X N(X',\theta)\,dX' \,. \tag{4}$$

The integrals represent the absorption factors to the depth X; $P(X',\theta)$ and $N(X',\theta)$ are the probability functions for absorption between depths X' and $X' + dX'$. Since the effectiveness of shielding is of concern, these probability functions for absorption are the quantities for which a description is sought.

Listing the transmission and reflection factors as a function of $X = Z/r_0$ reveals that these factors are very slowly varying functions of incident electron energy. For computational purposes, these factors can be considered independent of energy. The energy dependence of transmission and reflection is largely taken into account by the energy dependence of r_0. Because of this independence of transmission and reflection coefficients on E_0, it is reasonable to take coefficients averaged over all source energies in the range of interest. The electron absorption factors can then be described as functions of the variable X by an empirical curve fitted to Berger's data.

In a similar manner Burrell, et al., examined Berger's data for sources whose fluxes were normally incident by considering only the factors for $\theta = 0$ and isotropically incident by integrating the factors over θ between 0 and $\pi/2$. The results for energy and number absorption are listed below, since they will be used as a basis of comparison for the cosine law result.

<u>Normal Incidence</u>

$$\begin{aligned}\int_0^X P(X',0)\,dX' &= 1 - \left[T_E(X,0) + B_E(X,0)\right] \\ &= 0.95\left[1 - \exp\left(-0.653X - 2.40X^2 - 6.89X^3\right)\right] \,.\end{aligned} \tag{5}$$

$$\int_0^X N(X', 0)\, dX' = 1 - \left[T_N(X, 0) + B_N(X, 0)\right]$$

$$= 0.912 \left[1 - \exp\,(-0.0512X + 1.128X^2 - 9.38X^3\right]. \tag{6}$$

Isotropic Incidence

$$\int_0^{\pi/2} \int_0^X P(X', \theta)\, dX' \sin\theta \; d\theta = 1 - \left[T_E(X) + B_E(X)\right]$$

$$= 0.72 \left[1 - \exp(-2.47X + 0.752X^2 - 11.38X^3)\right]. \tag{7}$$

$$\int_0^{\pi/2} \int_0^X N(X', \theta)\, dX' \sin\theta \; d\theta = 1 - \left[T_N(X) + B_N(X)\right]$$

$$= 0.63 \left[1 - \exp(-0.152X + 0.479X^2 - 11.84X^3)\right]. \tag{8}$$

These expressions yield the desired probability functions for absorption by changing variables from X to Z/r_0 and differentiating the equations with respect to Z, as follows:

Normal Incidence

$$P(Z.0) = \frac{1}{r_0}\left[0.62 + 4.56\left(\frac{Z}{r_0}\right) + 19.64\left(\frac{Z}{r_0}\right)^2\right] \exp\left[-0.653\left(\frac{Z}{r_0}\right) - 2.40\left(\frac{Z}{r_0}\right)^2 - 6.89\left(\frac{Z}{r_0}\right)^3\right]. \tag{9}$$

$$N(Z,0) = \frac{1}{r_0}\left[0.467 - 2.057\left(\frac{Z}{r_0}\right) + 25.66\left(\frac{Z}{r_0}\right)^2\right] \exp\left[-0.0512\left(\frac{Z}{r_0}\right) + 1.128\left(\frac{Z}{r_0}\right)^2 - 9.38\left(\frac{Z}{r_0}\right)^3\right]. \qquad (10)$$

Isotropic Incidence

$$P(Z) = \frac{1}{r_0}\left[1.78 - 1.083\left(\frac{Z}{r_0}\right) + 24.58\left(\frac{Z}{r_0}\right)^2\right] \exp\left[-2.47\left(\frac{Z}{r_0}\right) + 0.752\left(\frac{Z}{r_0}\right)^2 - 11.38\left(\frac{Z}{r_0}\right)^3\right]. \qquad (11)$$

$$N(Z) = \frac{1}{r_0}\left[0.0958 - 0.604\left(\frac{Z}{r_0}\right) + 22.38\left(\frac{Z}{r_0}\right)^2\right] \exp\left[-0.152\left(\frac{Z}{r_0}\right) + 0.479\left(\frac{Z}{r_0}\right)^2 - 11.84\left(\frac{Z}{r_0}\right)^3\right]. \qquad (12)$$

ANALYSIS OF COSINE LAW SOURCE DATA

There is physical justification for assuming that a cosine current source represents a better approximation to reality than either of the normal or isotropic current sources previously examined. The penetration of an electron into a shield is a function of its angle of incidence. A spacecraft will be randomly oriented to the electron field along the trajectory of the spacecraft. It can be assumed that all angles of incidence between the incoming electrons and the shielding of the craft occur with equal probability. Thus, the time

average of the electron flux is isotropic to the surface of the shield. Consider a plane-parallel plate shield: the number of electrons that enter a unit area of the shield is proportional to the cosine of the angle between a perpendicular to the shield and the vector that represents the incident direction of the electron.

Berger and Seltzer [3] have further reported Monte Carlo energy transfer data for such a cosine law source at seven energies between 0.5 and 5.99 MeV as a function of the reduced foil thickness, x. The reflection data are limited to the maximum reflection coefficient that is a constant for $x \geq 0.3$. To obtain values of B_E for $x = 0.1$ and 0.2, the original Berger data at various angles were weighted by the cosine function, then integrated. The cosine law energy reflection coefficients are expressed:

$$B_E(x) = \frac{1}{2} \int_0^{\pi/2} B_E(x') \cos\theta \sin\theta \, d\theta \, . \tag{13}$$

Berger's data for number reflection displays considerable variation with incident energy E_0. Despite the fact that some inaccuracy does result by averaging over all energies, this method was used to retain the simple descriptive nature of the expressions for the number absorption case. This procedure reduces tables of data to a more convenient functional form. In the number transmission case, however, the evaluation of integrals analogous to equation (13) yielded values of B_N so large that, when they were added to Berger's values for T_N and the sum subtracted from one, they gave negative values for the absorption coefficients at $x = 0.1$ and 0.2. The creation of secondary or knock-on electrons is possible for thin shields of low atomic mass. The probability that an electron will cause secondaries increases with its incident energy, but the flux of electrons in space diminishes exponentially as the electron energies increase. Since aluminum has an intermediate atomic mass of 27 and the work is directed toward examination of thick shields, the possibility that significant numbers of secondary electrons would be encountered was discarded. Using this criteria, the values for B_N listed in Table I were chosen. The empirical fit to data using these values would appear reasonable.

TABLE I. ENERGY AND NUMBER REFLECTION COEFFICIENTS FOR COSINE LAW INCIDENCE[a]

$Z/r_0 = x$	B_E	B_N
0.1	0.087	0.100
0.2	0.110	0.150
0.3 or larger	0.122	0.200

a. Average values for all energies between 0 and 6 MeV.

Equations (14) through (17) were obtained for the energy and number absorption factors and probability density functions for a cosine law source.

Cosine Law Incidence

$$\int_0^{\pi/2}\int_0^{X} P(x', \theta)\, dx' \sin\theta\, d\theta = 1 - \left[T_E(x) + B_E(x)\right] \tag{14}$$

$$= 0.878\left[1 - \exp(-2.08x - 3.54x^2 - 6.08x^3)\right].$$

$$\int_0^{\pi/2}\int_0^{X} N(x', \theta)\, dx' \sin\theta\, d\theta = 1 - \left[T_N(x) + B_N(x)\right] \tag{15}$$

$$= 0.8\left[1 - \exp(-0.225x + 0.321x^2 - 6.30x^3)\right].$$

$$P(Z) = \frac{1}{r_0}\left[1.83 + 0.622\left(\frac{Z}{r_0}\right) + 16.01\left(\frac{Z}{r_0}\right)^2\right] \exp\left[-2.08\left(\frac{Z}{r_0}\right) - 0.354\left(\frac{Z}{r_0}\right)^2 - 6.08\left(\frac{Z}{r_0}\right)^3\right]. \tag{16}$$

$$N(Z) = \frac{1}{r_0}\left[0.180 - 0.497\left(\frac{Z}{r_0}\right) + 15.1\left(\frac{Z}{r_0}\right)^2\right] \exp\left[-0.225\left(\frac{Z}{r_0}\right) + 0.321\left(\frac{Z}{r_0}\right)^2 - 6.30\left(\frac{Z}{r_0}\right)^3\right] . \tag{17}$$

It should be noted that these curve fits are not least squares fits in the true sense. Rather, the least squares program devised by Watts and loaned to the author fits polynomial coefficients. In this work the data were in the form of x, the scaled target thickness versus y, the energy or number absorption, i.e.,

$$y_E(x) = 1 - \left[T_E(x) + B_E(x)\right] , \tag{18}$$

$$y_N(x) = 1 - \left[T_N(x) + B_N(x)\right] , \tag{19}$$

for energy and number absorption, respectively. Thus, in order to obtain a fit of the form

$$y = A\left[1 - \exp(Bx + Cx^2 + Dx^3)\right] , \tag{20}$$

equation (19) is manipulated to give:

$$\log\left(\frac{A - y}{A}\right) = Bx + Cx^2 + Dx^3 . \tag{21}$$

If the quantities on the left-hand side of this equation are fitted, the least squares program generates the coefficients B, C, and D. The coefficient A corresponds to the maximum value for the absorption factor, i.e.,

$$A = 1 - (T + B) , \tag{22}$$

when $T = 0$ and B equals its maximum value that is constant for $Z/r_0 \geq 0.3$.

DOSE RATE

The absorption probability functions for normal, isotropic, and cosine law sources, equations (9), (11), and (16), are dependent on Z the shield thickness and on E the incident energy through the intermediacy of r_0 the mean electron path length. The energy dependence of r_0 is given by Burrell, et al. [1] as

$$r_0 = (1.33 - 0.019E)\left(\sqrt{0.2713E^2 + 0.0121} - 0.11\right), \tag{23}$$

where the factor on the right is an empirical fit to the range data given in the tables by Berger and Seltzer [4].

The absorption probability function represents the probability that an electron of energy F has undergone absorption during passage through a shield of thickness Z. An example of the energy dependence of equations (9), (11), and (16) is given in Figure 1 for an arbitrarily chosen value of $Z = 0.2$ gm/cm^2. The effect of the value of the shield thickness Z on the absorption probability for the cosine distribution is shown in Figure 2.

The energy deposition or dose rate D(Z) obtained from an absorption probability function p(Z) for a given differential energy spectrum $\phi(E)$ incident on a shield is given by:

$$D(Z) = K \int_{E_{min}}^{10} E\,\phi(E)\; p(Z)\; dE. \tag{24}$$

K is a conversion constant which yields the dose in the units desired. E_{min} corresponds to the minimum electron energy necessary to penetrate a shield of thickness Z.

Figure 3 is a comparison of the dose as a function of shield thickness predicted by equation (24) for the normal, isotropic, and cosine law absorption probability functions. Curves are shown for two examples of idealized differential electron spectra.

$$\phi(E) = e^{-E} \ \frac{\text{electrons}}{\text{cm}^2\text{-MeV-sec}} \tag{25}$$

and

$$\phi(E) = 4.0\ e^{-4E} \ \frac{\text{electrons}}{\text{cm}^2\text{-MeV-sec}} \ . \tag{26}$$

The graph demonstrates that cosine incident current leads to a larger estimate of the dose than that from an isotropic source. This is as expected from the comparison of the probability functions in Figure 2.

The increase in the dose is significant for thick shields to the extent that the cosine source is believed to be a better approximation to physical conditions in space. If the electron flux is described by equation (25), the dose is as high as a factor of two greater from the cosine source as from an isotropic source.

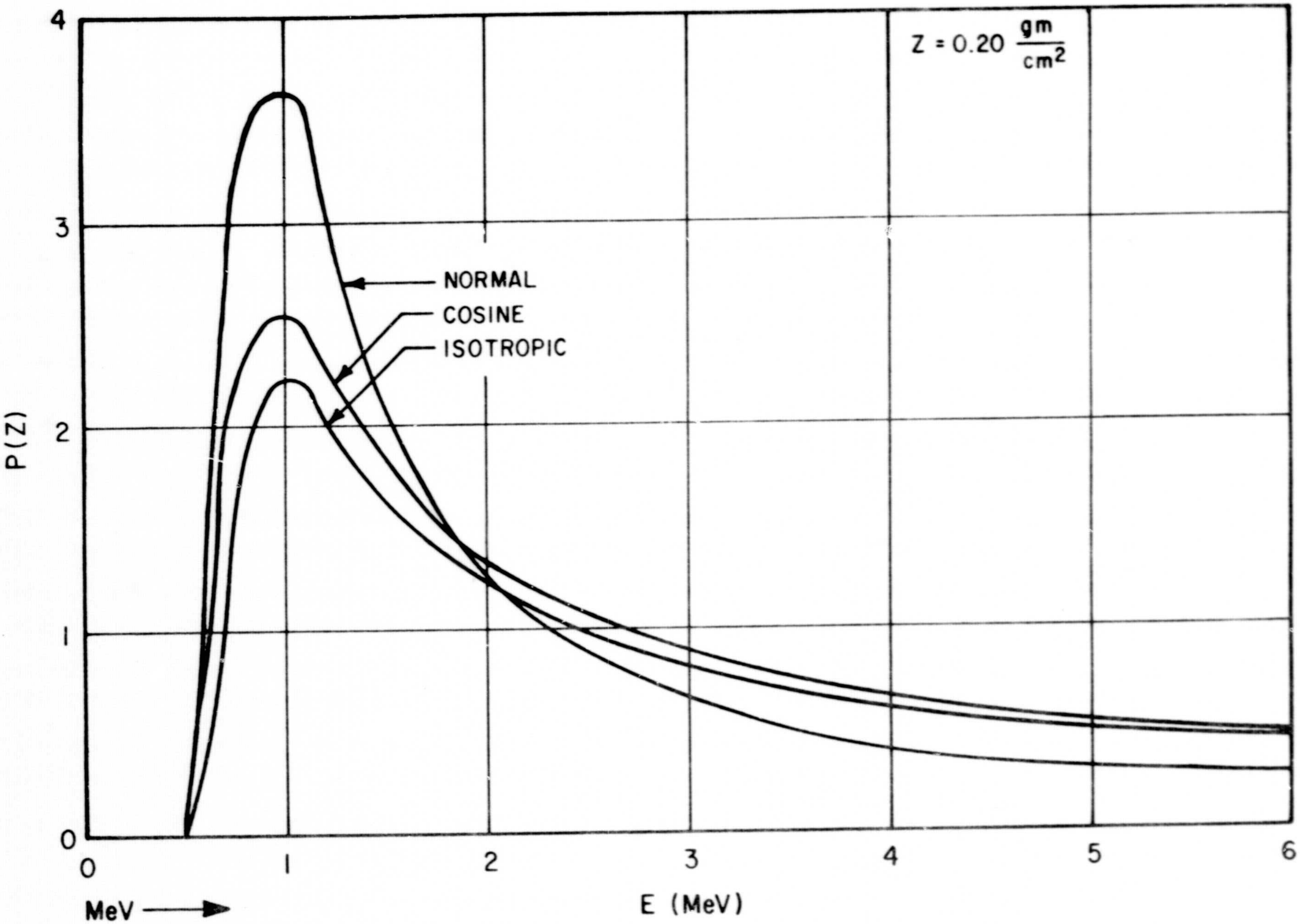

FIGURE 1. ENERGY DEPENDENCE OF THE ABSORPTION PROBABILITY FUNCTION FOR THREE DIFFERENT SOURCES IN A SHIELD OF ARBITRARY THICKNESS Z

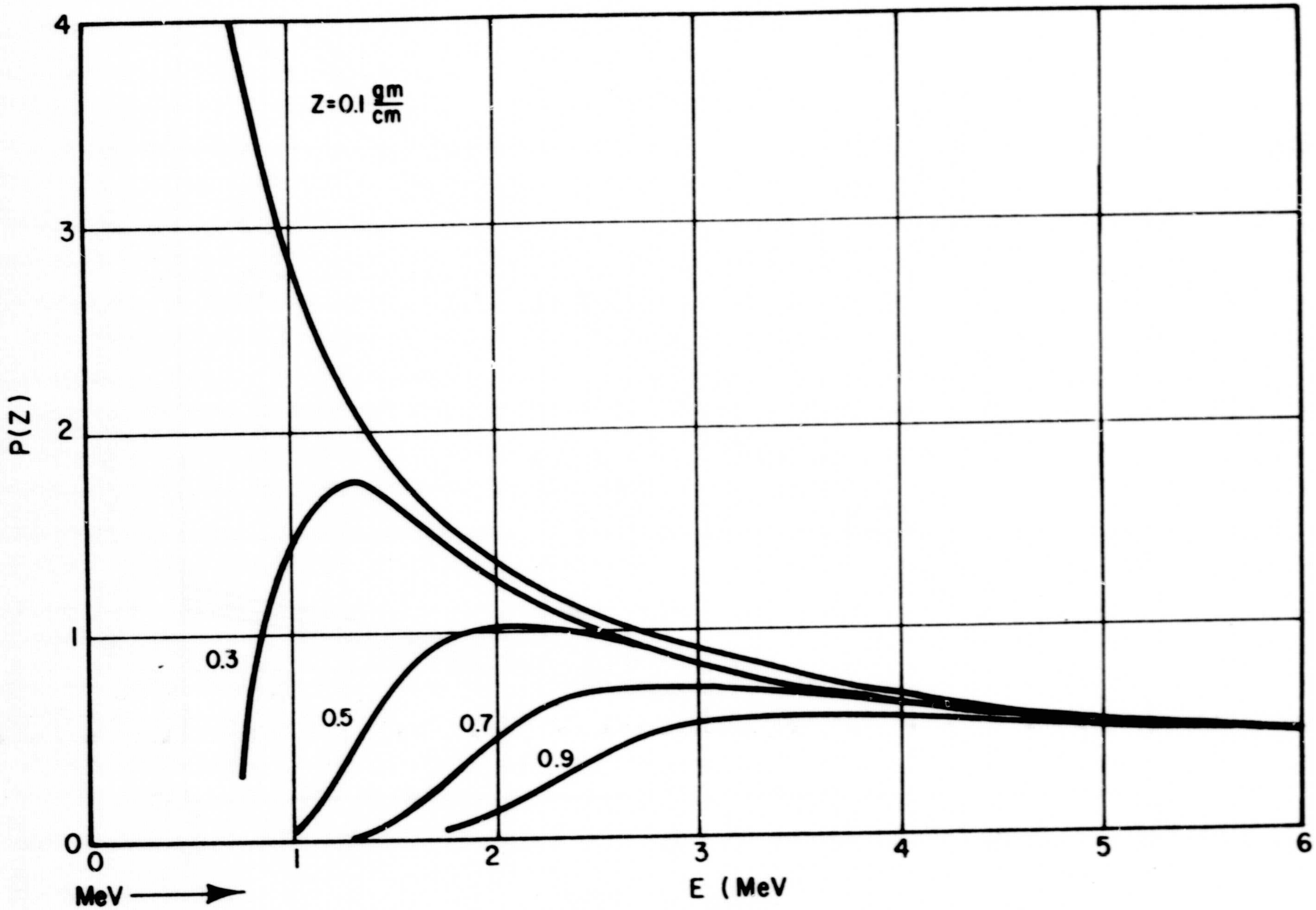

FIGURE 2. ABSORPTION PROBABILITY FUNCTION OF COSINE SOURCE FOR SEVERAL SHIELD THICKNESSES

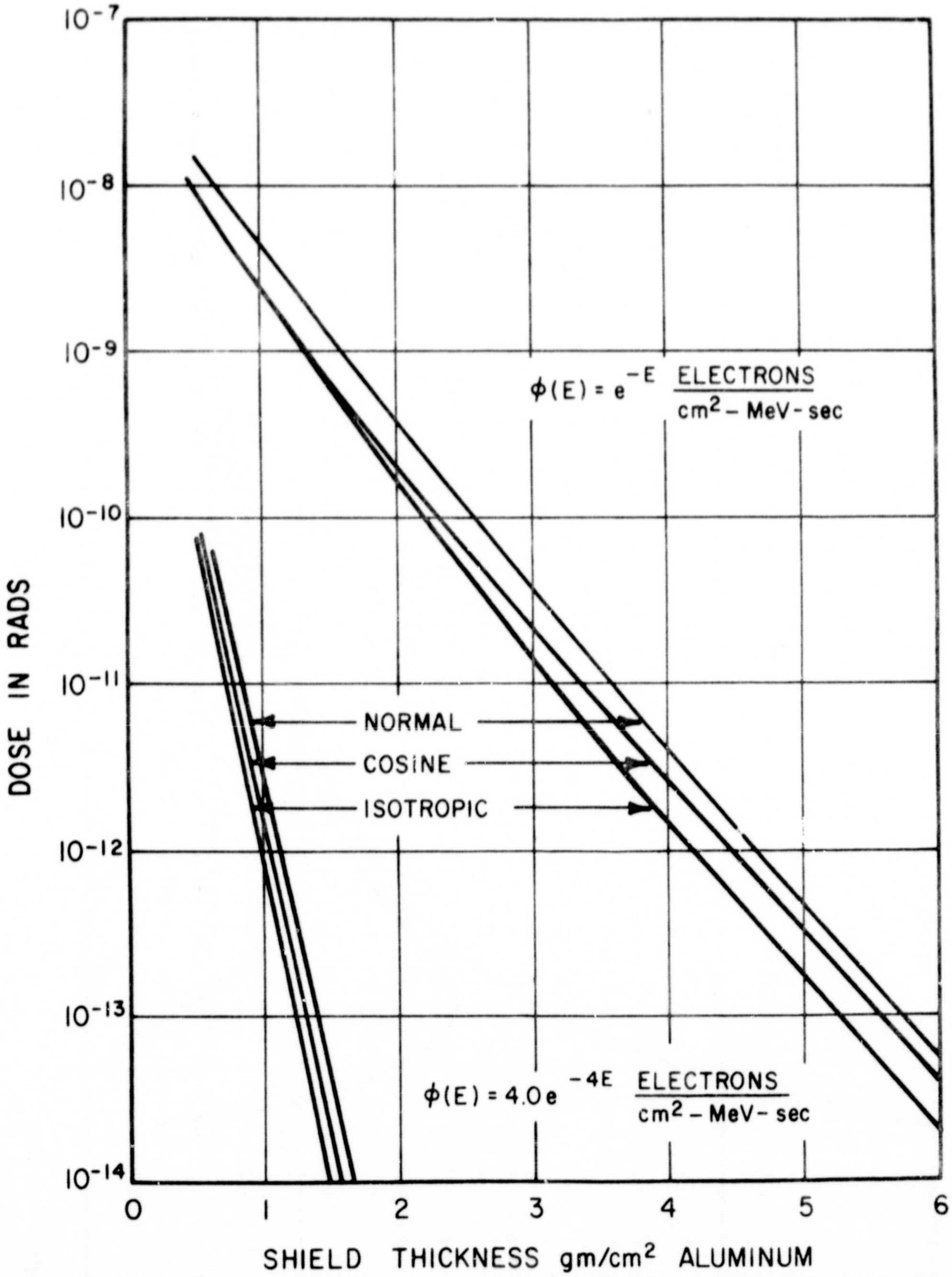

FIGURE 3. COMPARISON OF ELECTRON DOSE FROM THE THREE DIFFERENT SOURCES FOR TWO ELECTRON SPECTRA

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